Determine constants a, b, and c, so that the functions $y_0(t) = a$ and $y_1(t) = b + ct$ form an orthonormal set on the interval $0 \le t \le 1$. [104 中正化工 1]

[解] $\| y_0(t) \|^2 = 1 \Rightarrow \int_0^1 a \cdot a dt = 1 \Rightarrow a^2 t \Big|_0^1 = 1 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

$$\| y_1(t) \|^2 = 1 \Rightarrow \int_0^1 (b+ct)\cdot(b+ct) dt = 1 \Rightarrow (b^2 t + bct^2 + \frac{c^2 t^3}{3})\Big|_0^1 = 1 \Rightarrow b^2 + bc + \frac{c^2}{3} = 1 \cdots\cdots \text{(i)}$$

$$< y_0(t), y_1(t) > = 0 \Rightarrow \int_0^1 a \cdot (b+ct) dt = 0 \Rightarrow (abt + \frac{act^2}{2})\Big|_0^1 = 0 \Rightarrow ab + \frac{ac}{2} = 0 \cdots\cdots \text{(ii)}$$

$$a = 1\text{時}, \text{(i)}, \text{(ii)} \Rightarrow \begin{cases} b^2 + bc + \dfrac{c^2}{3} = 1 \\ b + \dfrac{c}{2} = 0 \end{cases} \Rightarrow c = -2b, b^2 - 2b^2 + \frac{4b^2}{3} = 1 \Rightarrow b = \pm\sqrt{3}, c = \mp 2\sqrt{3}$$

$$a = -1\text{時}, \text{(i)}, \text{(ii)} \Rightarrow \begin{cases} b^2 + bc + \dfrac{c^2}{3} = 1 \\ -b - \dfrac{c}{2} = 0 \end{cases} \Rightarrow c = -2b, b^2 - 2b^2 + \frac{4b^2}{3} = 1 \Rightarrow b = \pm\sqrt{3}, c = \mp 2\sqrt{3}$$