

Find the tangent to the ellipse $\frac{1}{4}x^2 + y^2 = 1$ at $P(\sqrt{2}, \frac{1}{\sqrt{2}})$. [100 彰師大積體電路 7]

[解] $\mathbf{r}(t) = 2\cos t \mathbf{i} + \sin t \mathbf{j}$, 由 $2\cos t = \sqrt{2}$, $\sin t = \frac{1}{\sqrt{2}} \Rightarrow P$ 點對應 $t = \frac{\pi}{4}$

$$\mathbf{r}'(t) = -2\sin t \mathbf{i} + \cos t \mathbf{j} \Rightarrow \mathbf{r}'(\pi/4) = -\sqrt{2}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} \Rightarrow \text{斜率為 } \frac{\frac{1}{\sqrt{2}}}{-\sqrt{2}} = -\frac{1}{2}$$

$$\text{切線為 } y - \frac{1}{\sqrt{2}} = -\frac{1}{2}(x - \sqrt{2}) \Rightarrow x + 2y = 2\sqrt{2}$$

