

Let $f(x, y) = e^{xy} \sin(x + y)$, (1) In what direction, starting at $(0, \pi/2)$, is f changing the fastest? (2) In what direction, starting at $(0, \pi/2)$, is f changing at 50% of its maximum rate? [99中山機電III2]

[解](1) f 在 $(0, \pi/2)$ 變化最快的方向為

$$\begin{aligned} \nabla f|_{(0, \pi/2)} &= \left(\frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} \right) \bigg|_{(0, \pi/2)} \\ &= e^{xy} \{ [y \sin(x + y) + \cos(x + y)] \mathbf{i} + [x \sin(x + y) + \cos(x + y)] \mathbf{j} \} \bigg|_{(0, \pi/2)} = \frac{\pi}{2} \mathbf{i} \end{aligned}$$

(2) f 在 $(0, \pi/2)$ 變化最快為 x 軸的方向， f 在 $(0, \pi/2)$ 變化為最快的 50% 與 x 軸夾 60° 或 120° ，

方向為 $\pm(\mathbf{i} + \sqrt{3}\mathbf{j})$ 或 $\pm(-\mathbf{i} + \sqrt{3}\mathbf{j})$