

求 $f(x) = \begin{cases} 1, & -1 < x < 1 \\ 0, & x < -1 \text{ 或 } x > 1 \end{cases}$ 之傅立葉積分式。[104 高第一機械一 9]

[解] 令 $f(x) = \int_0^\infty [a(\omega) \cos \omega x + b(\omega) \sin \omega x] d\omega$

$$a(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos \omega x dx = \frac{1}{\pi} \int_{-1}^1 1 \cdot \cos \omega x dx = \frac{1}{\pi \omega} [\sin(\omega) - \sin(-\omega)] = \frac{2 \sin \omega}{\pi \omega}$$

$$b(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin \omega x dx = 0$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \frac{\sin \omega}{\omega} \cos \omega x d\omega$$

Southern Taiwan University of Science and Technology