

Please find the ODE $(x+1)\frac{dy}{dx} + y = \ln x$, $y(1) = 10$. [104 中興精密 1(b)]

[解] 原式 $\Rightarrow y' + \frac{y}{x+1} = \frac{\ln x}{x+1}$

$$F = e^{\int \frac{1}{x+1} dx} = e^{\ln(x+1)} = x+1$$

$$y = \frac{1}{F} \left(\int F \cdot \frac{\ln x}{x+1} dx + C \right) = \frac{1}{x+1} \left(\int \ln x dx + C \right) = \frac{1}{x+1} (x \ln x - x + C)$$

$$y(1) = 10 \Rightarrow \frac{1}{2} (0 - 1 + C) = 10 \Rightarrow C = 21$$

$$y = \frac{1}{x+1} (x \ln x - x + 21)$$

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