Evaluate the determinant $\begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & 2 & 3 & 6 \end{vmatrix}$. [91 雲科大機械 5]

[解] $R_{41}(-2) \Rightarrow \begin{vmatrix} 0 & -4 & -5 & -12 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & 2 & 3 & 6 \end{vmatrix} = -1 \cdot \begin{vmatrix} -4 & -5 & -12 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix} \xrightarrow{R_{31}(4);\, R_{32}(-1)} = (-1) \cdot \begin{vmatrix} 0 & -9 & -12 \\ 0 & 2 & -1 \\ 1 & -1 & 0 \end{vmatrix}$

$= (-1) \cdot \begin{vmatrix} -9 & -12 \\ 2 & -1 \end{vmatrix} = (-1) \cdot (9 + 24) = -33$