Any real matrix **A** can be written as the sum of a symmetric **R** and a skew-symmetric matrix **S**. Now **A** is given below, what are **R** and **S**? [86 成大機械 4]

$$\mathrm{A} = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 0 & -1 \\ -3 & 13 & -4 \end{bmatrix}$$

[解] $\mathbf{A} = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 0 & -1 \\ -3 & 13 & -4 \end{bmatrix} \Rightarrow \mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 3 & 6 & -3 \\ -4 & 0 & 13 \\ -1 & -1 & -4 \end{bmatrix}$

$$\mathbf{A} = \frac{\mathbf{A}+\mathbf{A}}{2} + \frac{\mathbf{A}^{\mathrm{T}}-\mathbf{A}^{\mathrm{T}}}{2} = \frac{\mathbf{A}+\mathbf{A}^{\mathrm{T}}}{2} + \frac{\mathbf{A}-\mathbf{A}^{\mathrm{T}}}{2}$$

$$\mathbf{R} = \frac{\mathbf{A}+\mathbf{A}^{\mathrm{T}}}{2} = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 6 \\ -2 & 6 & -4 \end{bmatrix}, \mathbf{S} = \frac{\mathbf{A}-\mathbf{A}^{\mathrm{T}}}{2} = \begin{bmatrix} 0 & -5 & 1 \\ 5 & 0 & -7 \\ -1 & 7 & 0 \end{bmatrix}$$