

根據 Gram-Schmidt 正交化程序將下列三個行向量化為三個正交單位向量  $\mathbf{v}_1 = (2, 2, 1)$ ,  $\mathbf{v}_2 = (3, 6, 0)$ ,  $\mathbf{v}_3 = (-2, 7, -1)$ . [106 南台機械月考 3]

$$[\text{解}] \boldsymbol{\varphi}_1 = \mathbf{v}_1 \Rightarrow \mathbf{e}_1 = \frac{\boldsymbol{\varphi}_1}{|\boldsymbol{\varphi}_1|} = \frac{1}{3}(2, 2, 1)$$

$$\boldsymbol{\varphi}_2 = \mathbf{v}_2 - \langle \mathbf{v}_2, \mathbf{e}_1 \rangle \mathbf{e}_1 = (3, 6, 0) - \frac{6+12+0}{3} \cdot \frac{1}{3}(2, 2, 1) = (3, 6, 0) - 2(2, 2, 1) = (-1, 2, -2)$$

$$\mathbf{e}_2 = \frac{\boldsymbol{\varphi}_2}{|\boldsymbol{\varphi}_2|} = \frac{1}{3}(-1, 2, -2)$$

$$\begin{aligned} \boldsymbol{\varphi}_3 &= \mathbf{v}_3 - \langle \mathbf{v}_3, \mathbf{e}_1 \rangle \mathbf{e}_1 - \langle \mathbf{v}_3, \mathbf{e}_2 \rangle \mathbf{e}_2 = (-2, 7, -1) - \frac{-4+14-1}{3} \cdot \frac{1}{3}(2, 2, 1) - \frac{2+14+2}{3} \cdot \frac{1}{3}(-1, 2, -2) \\ &= (-2, 7, -1) - (2, 2, 1) - 2(-1, 2, -2) = (-2, 1, 2) \end{aligned}$$

$$\mathbf{e}_3 = \frac{\boldsymbol{\varphi}_3}{|\boldsymbol{\varphi}_3|} = \frac{1}{3}(-2, 1, 2)$$

$$\text{歸一正交基底為 } \mathbf{e}_1 = \frac{1}{3}(2, 2, 1), \mathbf{e}_2 = \frac{1}{3}(-1, 2, -2), \mathbf{e}_3 = \frac{1}{3}(-2, 1, 2)$$