$\mathbf{Ax}=\begin{bmatrix}1 & 2 & 0\\ 2 & 5 & 1\\ -1 & 3 & a\end{bmatrix}\mathbf{x}=\begin{bmatrix}3\\ 5\\ b\end{bmatrix}$, (a)If $\mathbf{x}$ has more than one solution, find a and b. (b)Find $\mathbf{x}_h$ and $\mathbf{x}_p$ such that $\mathbf{x}_g=\mathbf{x}_h+\mathbf{x}_p$ and $\mathbf{Ax}_g=\begin{bmatrix}3\\ 5\\ b\end{bmatrix}$, where $\mathbf{x}_g$, $\mathbf{x}_h$, and $\mathbf{x}_p$ are general solution, homogeneous solution, and particular solution, respectively. [105 中正機械 3]

[解]利用高斯消去法

$$\begin{bmatrix}1 & 2 & 0 & 3\\ 2 & 5 & 1 & 5\\ -1 & 3 & a & b\end{bmatrix}\xrightarrow{R_{12}(-2);\,R_{13}(1)}\begin{bmatrix}1 & 2 & 0 & 3\\ 0 & 1 & 1 & -1\\ 0 & 5 & a & b+3\end{bmatrix}\xrightarrow{R_{23}(-5)}\begin{bmatrix}1 & 2 & 0 & 3\\ 0 & 1 & 1 & -1\\ 0 & 0 & a-5 & b+8\end{bmatrix}$$

不只一解 $\Rightarrow a-5=0, b+8=0\Rightarrow a=5, b=-8$

令 $x_3=C$，代入第二式 $x_2+C=-1\Rightarrow x_2=-C-1$

代入第一式 $x_1+2(-C-1)=3\Rightarrow x_1=2C+5$

$$\mathbf{x}_g=\begin{bmatrix}2C+5\\ -C-1\\ C\end{bmatrix}=C\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}+\begin{bmatrix}5\\ -1\\ 0\end{bmatrix}\Rightarrow \mathbf{x}_h=C\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}, \mathbf{x}_p=\begin{bmatrix}5\\ -1\\ 0\end{bmatrix}$$