

Find the eigenvalues and the corresponding eigenvectors. $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ [104 逢甲電子 3]

$$[\text{解}] |\mathbf{A} - \lambda \mathbf{I}| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & -3-\lambda \end{vmatrix} = 0 \Rightarrow -(\lambda-1)(\lambda-2)(\lambda+3) = 0 \Rightarrow \lambda = 1, 2, -3$$

$$\lambda = 1, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 2, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = -3, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} 4 & 0 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \mathbf{x}_3 = \begin{bmatrix} -5 \\ -4 \\ 20 \end{bmatrix}$$