Calculate the inverse of the following matrix by Gauss-Jordan elimination. $\begin{bmatrix} -1 & 1 & 1 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$. [104 中正光電 2]

[解] $\begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_{12}(3);\, R_{13}(-1)} \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 3 & 1 & 0 \\ 0 & 2 & 3 & -1 & 0 & 1 \end{bmatrix}$

$\xrightarrow{R_1(-1);\, R_2(\frac{1}{2})} \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 2 & 3/2 & 1/2 & 0 \\ 0 & 2 & 3 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{R_{21}(1);\, R_{23}(-2)} \begin{bmatrix} 1 & 0 & 1 & 1/2 & 1/2 & 0 \\ 0 & 1 & 2 & 3/2 & 1/2 & 0 \\ 0 & 0 & -1 & -4 & -1 & 1 \end{bmatrix}$

$\xrightarrow{R_3(-1)} \begin{bmatrix} 1 & 0 & 1 & 1/2 & 1/2 & 0 \\ 0 & 1 & 2 & 3/2 & 1/2 & 0 \\ 0 & 0 & 1 & 4 & 1 & -1 \end{bmatrix} \xrightarrow{R_{31}(-1);\, R_{32}(-2)} \begin{bmatrix} 1 & 0 & 0 & -7/2 & -1/2 & 1 \\ 0 & 1 & 0 & -13/2 & -3/2 & 2 \\ 0 & 0 & 1 & 4 & 1 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 & 1 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -7/2 & -1/2 & 1 \\ -13/2 & -3/2 & 2 \\ 4 & 1 & -1 \end{bmatrix}$