Solve the following system using matrix technique. [103 虎尾動機 5]

$$\begin{cases} -x_1 + x_3 + x_4 + 2x_5 = 0 \\ x_2 + 3x_3 + 4x_5 = 0 \\ x_1 + 2x_2 + x_3 + x_4 + x_5 = 0 \\ -3x_1 + x_2 + 4x_5 = 0 \end{cases}$$

[解] $\begin{bmatrix} -1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 1 & 1 \\ -3 & 1 & 0 & 0 & 4 \end{bmatrix} \xrightarrow{R_{13}(1);\, R_{14}(-3)} \begin{bmatrix} -1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 & 4 \\ 0 & 2 & 2 & 2 & 3 \\ 0 & 1 & -3 & -3 & -2 \end{bmatrix}$

$\xrightarrow{R_{23}(-2);\, R_{24}(-1)} \begin{bmatrix} -1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & -4 & 2 & -5 \\ 0 & 0 & -6 & -3 & -6 \end{bmatrix} \xrightarrow{R_4(2)} \begin{bmatrix} -1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & -4 & 2 & -5 \\ 0 & 0 & -12 & -6 & -12 \end{bmatrix}$

$\xrightarrow{R_{34}(-3)} \begin{bmatrix} -1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & -4 & 2 & -5 \\ 0 & 0 & 0 & -12 & 3 \end{bmatrix}$

令 $x_4 = C \Rightarrow x_5 = 4C,\ x_3 = -\frac{9}{2}C,\ x_2 = -\frac{5}{2}C,\ x_1 = \frac{9}{2}C$