

Consider the matrix  $\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix}$ . (1) Find the eigenvalues and eigenvectors. (2) Compute  $\mathbf{A}^{18}$ .

[103 台科大自控 4]

$$\text{[解]} \quad |\mathbf{A} - \lambda \mathbf{I}| = 0 \Rightarrow \begin{vmatrix} -1-\lambda & 0 \\ 1 & -5-\lambda \end{vmatrix} = 0 \Rightarrow (\lambda+1)(\lambda+5) = 0 \Rightarrow \lambda = -1, -5$$

$$\lambda = -1, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} 0 & 0 \\ 1 & -4 \end{bmatrix} \mathbf{x} = 0 \Rightarrow \mathbf{x}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\lambda = -5, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} 4 & 0 \\ 1 & 0 \end{bmatrix} \mathbf{x} = 0 \Rightarrow \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\text{得 } \mathbf{D} = \begin{bmatrix} -1 & 0 \\ 0 & -5 \end{bmatrix}, \mathbf{S} = \begin{bmatrix} 4 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \mathbf{S}^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A}^{18} &= \mathbf{S} \mathbf{D}^{18} \mathbf{S}^{-1} = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} (-1)^{18} & 0 \\ 0 & (-5)^{18} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 1 & 5^{18} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 1-5^{18} & 4 \cdot 5^{18} \end{bmatrix} \end{aligned}$$