

The eigenvalues of matrix $\mathbf{B} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ are 3, 2 and 0. Find a matrix \mathbf{P} such that $\mathbf{P}^t \mathbf{B} \mathbf{P}$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ where } \mathbf{P}^t \text{ is the transpose of matrix } \mathbf{P}. \text{ [103 台科大機械 3(b)]}$$

$$\text{[解]} \lambda = 3, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \mathbf{x} = 0 \Rightarrow \mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \mathbf{e}_1 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 2, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x} = 0 \Rightarrow \mathbf{x}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \mathbf{e}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \mathbf{x} = 0 \Rightarrow \mathbf{x}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \Rightarrow \mathbf{e}_3 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\text{得 } \mathbf{P} = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$