Compute the eigenvalues of matrix $\mathbf{A}=\begin{bmatrix} 2 & -3 & -1 \\ -5 & 4 & 2 \\ 1 & -5 & -1 \end{bmatrix}$. [103 台科大機械 3(a)]

[解] $|\mathbf{A}-\lambda\mathbf{I}|=0 \Rightarrow \begin{vmatrix} 2-\lambda & -3 & -1 \\ -5 & 4-\lambda & 2 \\ 1 & -5 & -1-\lambda \end{vmatrix}=0$

$$-(\lambda-2)(\lambda-4)(\lambda+1)-25-6-(\lambda-4)-10(\lambda-2)+15(\lambda+1)=0$$

$$-(\lambda^3-5\lambda^2+2\lambda+8)-31+4\lambda+39=0 \Rightarrow \lambda^3-5\lambda^2-2\lambda=0 \Rightarrow \lambda=0,\ \frac{5\pm\sqrt{33}}{2}$$