

Find the directional derivative of $f = x^2 + y^2$ at the point P(1, 1) in the direction of the vector $\mathbf{a} = 2\mathbf{i} - 6\mathbf{j}$. [104 中正光機電 5]

$$[\text{解}] \quad \text{令 } f = x^2 + y^2 \Rightarrow \nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} = 2x\mathbf{i} + 2y\mathbf{j} \Rightarrow \nabla f|_{(1,1)} = 2\mathbf{i} + 2\mathbf{j}$$

$$\text{所求為 } \nabla f|_{(1,1)} \cdot \frac{\mathbf{a}}{|\mathbf{a}|} = (2\mathbf{i} + 2\mathbf{j}) \cdot \frac{2\mathbf{i} - 6\mathbf{j}}{\sqrt{2^2 + (-6)^2}} = \frac{-4}{\sqrt{10}}$$