Find the solution of the equation $(1+x^2)(dy-dx)=2xydx$ for which $y=1$ when $x=0$. [99 高師大電子 1]

[解]原式 $\Rightarrow dy-dx=\dfrac{2xy}{1+x^2}dx \Rightarrow y'-1=\dfrac{2xy}{1+x^2} \Rightarrow y'-\dfrac{2x}{1+x^2}y=1$

$$F=e^{\int -\frac{2x}{1+x^2}dx}=e^{-\ln(1+x^2)}=\frac{1}{1+x^2}$$

$$y=\frac{1}{F}[\int F\cdot 1dx+C]=(1+x^2)[\int \frac{1}{1+x^2}\cdot 1dx+C]=(1+x^2)[\tan^{-1}x+C]$$