Find the general solution of $y'' + 2y' + y = 3e^{-x}$. [99 交大機械甲 3]

[解]特徵方程式為

$\lambda^2 + 2\lambda + 1 = 0 \Rightarrow \lambda = -1, -1$

$y_h(x) = (C_1 + C_2 x)e^{-x}$

Let $y_p(x) = Ax^2 e^{-x} \Rightarrow y_p' = (-Ax^2 + 2Ax)e^{-x} \Rightarrow y_p'' = (Ax^2 - 4Ax + 2A)e^{-x}$

代入原式得

$(Ax^2 - 4Ax + 2A)e^{-x} + 2(-Ax^2 + 2Ax)e^{-x} + Ax^2 e^{-x} = 3e^{-x} \Rightarrow A = \dfrac{3}{2}$

解為 $y(x) = y_h(x) + y_p(x) = (C_1 + C_2 x + \dfrac{3}{2} x^2)e^{-x}$