

Solve $(e^{2y} - y \cos xy)dx + (2xe^{2y} - x \cos xy + 2y)dy = 0$. [97 高師大光電 1]

$$[\text{解}] M = e^{2y} - y \cos xy \Rightarrow \frac{\partial M}{\partial y} = 2e^{2y} - \cos xy + xy \sin xy$$

$$N = 2xe^{2y} - x \cos xy + 2y \Rightarrow \frac{\partial N}{\partial x} = 2e^{2y} - \cos xy + xy \sin xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{原式為正合微分方程式}$$

$$u = \int_x M dx + f(y) = \int_x (e^{2y} - y \cos xy) dx + f(y) = xe^{2y} - \sin xy + f(y)$$

$$\frac{\partial u}{\partial y} = N \Rightarrow 2xe^{2y} - x \cos xy + f'(y) = 2xe^{2y} - x \cos xy + 2y$$

$$f'(y) = 2y \Rightarrow f(y) = y^2$$

$$\text{解為 } u = C \Rightarrow xe^{2y} - \sin xy + y^2 = C$$