求解微分方程式$(x^2+4xy)dx+(2x^2+y)dy=0$. [97 海洋運航 1]

[解]$M=x^2+4xy \Rightarrow \frac{\partial M}{\partial y}=4x$

$N=2x^2+y \Rightarrow \frac{\partial N}{\partial x}=4x$

$\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y} \Rightarrow$原式為正合微分方程式

$u=\int_x M\,dx+k(y)=\int_x (x^2+4xy)\,dx+k(y)=x^3/3+2x^2y+k(y)$

$\frac{\partial u}{\partial y}=N \Rightarrow 2x^2+k'(y)=2x^2+y \Rightarrow k'(y)=y \Rightarrow k(y)=y^2/2$

解為 $x^3/3+2x^2y+y^2/2=C$