Please reduce the second-order differential equation $yy'' = 2y'^2$ to first order, and solve the equation. [96 中興材料 1]

[解]令 $p = y' \Rightarrow y'' = \dfrac{dy'}{dx} = \dfrac{dy'}{dy}\dfrac{dy}{dx} = p\dfrac{dp}{dy}$

原式 $\Rightarrow y \cdot p\dfrac{dp}{dy} = 2p^2 \Rightarrow \dfrac{dp}{p} = \dfrac{2dy}{y}$ 此為 $p$ 對 $y$ 的一階微分方程式

$\displaystyle\int \dfrac{dp}{p} = \int \dfrac{2dy}{y} + k \Rightarrow \ln p = 2\ln y + k \Rightarrow \ln\dfrac{p}{y^2} = k \Rightarrow \dfrac{p}{y^2} = C_1 \Rightarrow \dfrac{\frac{dy}{dx}}{y^2} = C_1 \Rightarrow \dfrac{dy}{y^2} = C_1 dx$

$\displaystyle\int \dfrac{dy}{y^2} = \int C_1 dx + C_2 \Rightarrow -\dfrac{1}{y} = C_1 x + C_2 \Rightarrow y = -\dfrac{1}{C_1 x + C_2}$