

Solve the differential equation $y' = \frac{3x \sin 2y - 2y}{x - 2x^2 \cos 2y}$. [90 清大材料 1]

[解]原式 $\Rightarrow (3x \sin 2y - 2y)dx + (2x^2 \cos 2y - x)dy = 0$

$$M = 3x \sin 2y - 2y \Rightarrow \frac{\partial M}{\partial y} = 6x \cos 2y - 2$$

$$N = 2x^2 \cos 2y - x \Rightarrow \frac{\partial N}{\partial x} = 4x \cos 2y - 1$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{(6x \cos 2y - 2) - (4x \cos 2y - 1)}{2x^2 \cos 2y - x} = \frac{2x \cos 2y - 1}{x(2x \cos 2y - 1)} = \frac{1}{x}$$

$$\mu = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$u = \int_x \mu M dx + f(y) = \int_x x(3x \sin 2y - 2y) dx + f(y)$$

$$= x^3 \sin 2y - x^2 y + f(y)$$

$$\frac{\partial u}{\partial y} = \mu N \Rightarrow 2x^3 \cos 2y - x^2 + f'(y) = x(2x^2 \cos 2y - x) \Rightarrow f'(y) = 0 \Rightarrow f(y) = 0$$

$$\text{解 } u = C \text{ 為 } x^3 \sin 2y - x^2 y = C$$