Apply the power series method to solve the following equation $y' = y$. [90 暨南電子 5]

[解]設 $y = \sum_{m=0}^{\infty} A_m x^m \Rightarrow y' = \sum_{m=0}^{\infty} m A_m x^{m-1}$

原式 $\Rightarrow \sum_{m=0}^{\infty} m A_m x^{m-1} = \sum_{m=0}^{\infty} A_m x^m \Rightarrow \sum_{m=1}^{\infty} m A_m x^{m-1} - \sum_{m=0}^{\infty} A_m x^m = 0$

$$\sum_{m=1}^{\infty} m A_m x^{m-1} - \sum_{m=1}^{\infty} A_{m-1} x^{m-1} = 0 \Rightarrow \sum_{m=1}^{\infty} (m A_m - A_{m-1}) x^{m-1} = 0$$

$$m A_m - A_{m-1} = 0 \Rightarrow A_m = \frac{A_{m-1}}{m}, \quad m = 1, 2, 3, \cdots$$

$A_1 = \frac{A_0}{1}, A_2 = \frac{A_1}{2} = \frac{A_0}{1 \cdot 2}, A_3 = \frac{A_2}{3} = \frac{A_0}{1 \cdot 2 \cdot 3}, \cdots \Rightarrow$ 知 $A_m = \frac{A_0}{m!}$

$$y = A_0 \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) = A_0 e^x$$