

$$y'' - 2y' + 2y = e^x \cos x. \text{ [90 北科大自動化甲(A)(3)]}$$

[解] The characteristic equation is

$$\lambda^2 - 2\lambda + 2 = 0 \Rightarrow \lambda = 1 \pm i$$

$$y_h(x) = e^x(C_1 \cos x + C_2 \sin x)$$

$$\text{Let } y_p(x) = e^x(Ax \sin x + Bx \cos x) \Rightarrow y_p' = e^x[(Ax - Bx + A) \sin x + (Ax + Bx + B) \cos x]$$

$$y_p'' = e^x[(-2Bx + 2A - 2B) \sin x + (2Ax + 2A + 2B) \cos x]$$

代入原式得

$$e^x[(-2Bx + 2A - 2B) \sin x + (2Ax + 2A + 2B) \cos x] - 2e^x[(Ax - Bx + A) \sin x + (Ax + Bx + B) \cos x] + 2e^x(Ax \sin x + Bx \cos x) = e^x \cos x$$

$$e^x(-2B \sin x + 2A \cos x) = e^x \cos x \Rightarrow A = \frac{1}{2}, B = 0$$

$$\text{解為 } y(x) = y_h(x) + y_p(x) = e^x(C_1 \cos x + C_2 \sin x + \frac{1}{2} x \sin x)$$