解微分方程式 $y' = 2(y\sin 2x + \cos 2x)/\cos 2x$。[90 北科大自動化(A)(1)]

[解]原式 $\Rightarrow 2(y\sin 2x + \cos 2x)dx - \cos 2x dy = 0$

$M = 2(y\sin 2x + \cos 2x) \Rightarrow \dfrac{\partial M}{\partial y} = 2\sin 2x$

$N = -\cos 2x \Rightarrow \dfrac{\partial N}{\partial x} = 2\sin 2x$

$\dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial y} \Rightarrow$原式為正合微分方程式

$u = \int_x M\,dx + k(y) = \int_x 2(y\sin 2x + \cos 2x)\,dx + k(y) = -y\cos 2x + \sin 2x + k(y)$

$\dfrac{\partial u}{\partial y} = N \Rightarrow -\cos 2x + k'(y) = -\cos 2x \Rightarrow k'(y) = 0 \Rightarrow k(y) = 0$

解為 $-y\cos 2x + \sin 2x = C$

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