Find the general solution of the differential equation $xy'+2y=e^{x^2}$. [85 交大機械乙 1(a)]

[解]原式 $\Rightarrow y'+\frac{2}{x}y=\frac{e^{x^2}}{x}$

$$F=e^{\int \frac{2}{x}dx}=e^{2\ln x}=x^2$$

$$y=\frac{1}{F}(\int F\cdot\frac{e^{x^2}}{x}dx+C)=\frac{1}{x^2}(\int x^2\cdot\frac{e^{x^2}}{x}dx+C)=\frac{1}{x^2}(\int xe^{x^2}dx+C)$$

$$=\frac{1}{x^2}(\frac{e^{x^2}}{2}+C)=\frac{e^{x^2}}{2x^2}+\frac{C}{x^2}$$