

Find the general solution of the differential equation $xy' + 2y = e^{x^2}$. [85 交大機械乙 1(a)]

$$\text{[解]原式} \Rightarrow y' + \frac{2}{x}y = \frac{e^{x^2}}{x}$$

$$F = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = x^2$$

$$y = \frac{1}{F} \left(\int F \cdot \frac{e^{x^2}}{x} dx + C \right) = \frac{1}{x^2} \left(\int x^2 \cdot \frac{e^{x^2}}{x} dx + C \right) = \frac{1}{x^2} \left(\int x e^{x^2} dx + C \right)$$

$$= \frac{1}{x^2} \left(\frac{e^{x^2}}{2} + C \right) = \frac{e^{x^2}}{2x^2} + \frac{C}{x^2}$$

Southern Taiwan University of Science and Technology