

One solution of the ODE $x^2y'' - x^2y' - 2xy' + xy + 2y = 0$ is $y_1 = x$. Find the general solution. [106 暨南電機 5]

[解] 令 $y = vx \Rightarrow y' = v'x + v, y'' = v''x + 2v'$

$$\text{原式} \Rightarrow x^2(v''x + 2v') - (x^2 + 2x)(v'x + v) + (x + 2)vx = 0$$

$$x^3v'' + [2x^2 - (x^2 + 2x)x]v' + [-(x^2 + 2x) + (x + 2)x]v = 0$$

$$x^3v'' - x^3v' = 0 \Rightarrow v'' - v' = 0 \text{ 這是 } v' \text{ 的一階線性}$$

$$v' = C_1 e^{\int 1 dx} = C_1 e^x$$

$$v = \int C_1 e^x dx + C_2 = C_1 e^x + C_2$$

$$y = vx = C_1 x e^x + C_2 x$$

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