One solution of the ODE $x^2y'' - x^2y' - 2xy' + xy + 2y = 0$ is $y_1 = x$. Find the general solution. [106 暨南電機 5]

[解]令 $y = vx \Rightarrow y' = v'x + v,\ y'' = v''x + 2v'$

原式 $\Rightarrow x^2(v''x + 2v') - (x^2 + 2x)(v'x + v) + (x + 2)vx = 0$

$x^3v'' + [2x^2 - (x^2 + 2x)x]v' + [-(x^2 + 2x) + (x + 2)x]v = 0$

$x^3v'' - x^3v' = 0 \Rightarrow v'' - v' = 0$ 這是 v'的一階線性

$$v' = C_1 e^{\int 1 \cdot dx} = C_1 e^x$$

$$v = \int C_1 e^x dx + C_2 = C_1 e^x + C_2$$

$$y = vx = C_1 x e^x + C_2 x$$