Solve $xy' = (x+1)y - e^{-x}y^2$. [106 暨南電機 2]

[解]原式 $\Rightarrow y^{-2}y' - \dfrac{x+1}{x}y^{-1} = -\dfrac{e^{-x}}{x}$ ········(i)

$\Leftrightarrow u = y^{-1} \Rightarrow u' = -y^{-2}y'$

(i) $\Rightarrow -u' - \dfrac{x+1}{x}u = -\dfrac{e^{-x}}{x} \Rightarrow u' + \dfrac{x+1}{x}u = \dfrac{e^{-x}}{x}$

$F = e^{\int \frac{x+1}{x}dx} = e^{x+\ln x} = xe^x$

$u = \dfrac{1}{xe^x}[\int xe^x \cdot \dfrac{e^{-x}}{x}dx + C] = \dfrac{1}{xe^x}[x+C] = \dfrac{x+C}{xe^x}$

$y = u^{-1} = \dfrac{xe^x}{x+C}$