

Solve $xy' = (x+1)y - e^{-x}y^2$. [106 暨南電機 2]

[解]原式 $\Rightarrow y^{-2}y' - \frac{x+1}{x}y^{-1} = -\frac{e^{-x}}{x}$(i)

令 $u = y^{-1} \Rightarrow u' = -y^{-2}y'$

(i) $\Rightarrow -u' - \frac{x+1}{x}u = -\frac{e^{-x}}{x} \Rightarrow u' + \frac{x+1}{x}u = \frac{e^{-x}}{x}$

$F = e^{\int \frac{x+1}{x} dx} = e^{x+\ln x} = xe^x$

$u = \frac{1}{xe^x} [\int xe^x \cdot \frac{e^{-x}}{x} dx + C] = \frac{1}{xe^x} [x + C] = \frac{x+C}{xe^x}$

$y = u^{-1} = \frac{xe^x}{x+C}$

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