Solve the ODE by variable substitution $x^2y'' - 5xy' + 8y = 2\ln x$. [106 成大機械 1]

[解]令 $x = e^t \Rightarrow t = \ln x$ 原式為

$$\frac{d}{dt}\left(\frac{d}{dt} - 1\right)y - 5\frac{dy}{dt} + 8y = 2t \Rightarrow \frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = 2t \cdots\cdots\cdots\text{(i)}$$

特徵方程式 $\lambda^2 - 6\lambda + 8 = 0 \Rightarrow \lambda = 2, 4 \Rightarrow y_h = C_1e^{2t} + C_2e^{4t}$

令 $y_p = At + B \Rightarrow y_p' = A \Rightarrow y_p'' = 0$

$$\text{(i)} \Rightarrow 0 - 6A + 8(At + B) = 2t \Rightarrow A = \frac{1}{4},\ B = \frac{3}{16}$$

$$y = y_h + y_p = C_1e^{2t} + C_2e^{4t} + \frac{1}{4}t + \frac{3}{16}$$