Solve the first order ordinary differential equation $\dfrac{dy}{dx}=\dfrac{2(x^3+xy)}{x^2-y}$. [106 台科大機械 1(b)]

[解]原式$\Rightarrow y'(x^2-y)-2(x^3+xy)=0\cdots\cdots\cdots$(i)

各項權值分別為$(m-1)+2,\ (m-1)+m,\ 3,\ 1+m$, 得 $m=2$

令 $y=ux^2\Rightarrow y'=u'x^2+2ux$

$$(\text{i})\Rightarrow(u'x^2+2ux)(x^2-ux^2)-2(x^3+x\cdot ux^2)=0$$

$$(u'x^4-u'ux^4+2ux^3-2u^2x^3)-2(x^3+ux^3)=0\Rightarrow u'x-u'ux-2u^2-2=0$$

$$u'x(1-u)-2(u^2+1)=0\Rightarrow u'x(u-1)+2(u^2+1)=0$$

$$\frac{du}{dx}x(u-1)+2(u^2+1)=0\Rightarrow\frac{u-1}{2(u^2+1)}du+\frac{dx}{x}=0\Rightarrow\frac{1}{2}\int\frac{u-1}{u^2+1}du+\int\frac{dx}{x}=k$$

$$\frac{1}{2}\int\left(\frac{u}{u^2+1}-\frac{1}{u^2+1}\right)du+\int\frac{dx}{x}=k\Rightarrow\frac{1}{2}\left[\frac{1}{2}\ln(u^2+1)-\tan^{-1}u\right]+\ln x=k$$

$$\ln(u^2+1)-2\tan^{-1}u+4\ln x=4k\Rightarrow\ln[x^4(u^2+1)]-2\tan^{-1}u=C$$

$$\ln(y^2+x^4)-2\tan^{-1}\frac{y}{x^2}=C$$