

解微分方程式 $(2x + \frac{1}{y} - \frac{y}{x^2})dx + (2y + \frac{1}{x} - \frac{x}{y^2})dy = 0$. [106 台大生機 1]

$$[\text{解}] M = 2x + \frac{1}{y} - \frac{y}{x^2} \Rightarrow \frac{\partial M}{\partial y} = -\frac{1}{y^2} - \frac{1}{x^2}$$

$$N = 2y + \frac{1}{x} - \frac{x}{y^2} \Rightarrow \frac{\partial N}{\partial x} = -\frac{1}{x^2} - \frac{1}{y^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{原式為正合微分方程式}$$

$$u = \int_x M dx + f(y) = \int_x (2x + \frac{1}{y} - \frac{y}{x^2}) dx + f(y) = x^2 + \frac{x}{y} + \frac{y}{x} + f(y)$$

$$\frac{\partial u}{\partial y} = N \Rightarrow -\frac{x}{y^2} + \frac{1}{x} + f'(y) = 2y + \frac{1}{x} - \frac{x}{y^2} \Rightarrow f'(y) = 2y \Rightarrow f(y) = y^2$$

$$\text{解為 } u = C \Rightarrow x^2 + \frac{x}{y} + \frac{y}{x} + y^2 = C$$