Find the general solution to the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^3y^3$. [106 台大海洋丙四]

[解]原式 $\Rightarrow y^{-3}y'+\frac{1}{x}y^{-2}=x^3\cdots\cdots\cdots(\text{i})$

令 $u=y^{-2}\Rightarrow u'=-2y^{-3}y'$

$(\text{i})\Rightarrow -\frac{1}{2}u'+\frac{1}{x}u=x^3\Rightarrow u'-\frac{2}{x}u=-2x^3$

$F=e^{\int -\frac{2}{x}dx}=e^{-2\ln x}=x^{-2}$

$u=\frac{1}{x^{-2}}[\int x^{-2}\cdot(-2x^3)dx+C]=x^2[(-x^2)+C]=-x^4+Cx^2$

$y^{-2}=-x^4+Cx^2$