解 $x^2y'' + 6xy' + 6y = (20\ln x - 11)x^2$。[106 南台車輛期末考 2]

[解]令 $x = e^t \Rightarrow t = \ln x$，原式為

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = (20t - 11)e^{2t} \cdots\cdots\cdots(i)$$

(i)的特徵方程式 $\lambda^2 + 5\lambda + 6 = 0 \Rightarrow (\lambda + 2)(\lambda + 3) = 0 \Rightarrow \lambda = -2, -3$

$$y_h = C_1e^{-2t} + C_2e^{-3t}$$

$$\Rightarrow y_p = (At + B)e^{2t} \Rightarrow \frac{dy_p}{dt} = [2At + (A + 2B)]e^{2t} \Rightarrow \frac{d^2y_p}{dt^2} = [4At + (4A + 4B)]e^{2t}$$

$$(i) \Rightarrow [4At + (4A + 4B)]e^{2t} + 5[2At + (A + 2B)]e^{2t} + 6(At + B)e^{2t} = (20t - 11)e^{2t}$$

$$[20At + (9A + 20B)]e^{2t} = (20t - 11)e^{2t} \Rightarrow \begin{cases} 20A = 20 \\ 9A + 20B = -11 \end{cases} \Rightarrow A = 1, B = -1$$

$$y = y_h + y_p = C_1e^{-2t} + C_2e^{-3t} + (t - 1)e^{2t} = C_1x^{-2} + C_2x^{-3} + (\ln x - 1)x^2$$