Solve the ODE $y'' + y' - 12y = (-6x + 11)e^{2x}$. [106 南台車輛月考 4]

[解]特徵方程式 $\lambda^2 + \lambda - 12 = 0 \Rightarrow (\lambda + 4)(\lambda - 3) = 0 \Rightarrow \lambda = -4, 3$

$y_h = C_1 e^{-4x} + C_2 e^{3x}$

Let $y_p = (Ax + B)e^{2x} \Rightarrow y_p' = [2Ax + (A + 2B)]e^{2x}$

$y_p'' = [4Ax + (4A + 4B)]e^{2x}$

代入原式

$[4Ax + (4A + 4B)]e^{2x} + [2Ax + (A + 2B)]e^{2x} - 12(Ax + B)e^{2x} = (-6x + 11)e^{2x}$

$[-6Ax + (5A - 6B)]e^{2x} = (-6x + 11)e^{2x} \Rightarrow A = 1, B = -1$

得 $y(x) = y_h + y_p = C_1 e^{-4x} + C_2 e^{3x} + (x - 1)e^{2x}$