Please find the solution of $y'' - 2y' + y = 2\sin3x$, $y(0) = 2$, $y'(0)1$. [106 中興電機乙丙丁光電 8(2)]

[解]特徵方程式 $\lambda^2 - 2\lambda + 1 = 0 \Rightarrow (\lambda - 1)^2 = 0 \Rightarrow \lambda = 1, 1$

$y_h = (C_1 + C_2 x)e^x$

Let $y_p = A\sin3x + B\cos3x \Rightarrow y_p' = 3A\cos3x - 3B\sin3x \Rightarrow y_p'' = -9A\sin3x - 9B\cos3x$

代入原式

$(-9A\sin3x - 9B\cos3x) - 2(3A\cos3x - 3B\sin3x) + (A\sin3x + B\cos3x) = 2\sin3x$

$(-8A + 6B)\sin3x + (-6A - 8B)\cos3x = 2\sin3x$

$\begin{cases} -8A + 6B = 2 \\ -6A - 8B = 0 \end{cases} \Rightarrow A = -\dfrac{4}{25}, B = \dfrac{3}{25}$

得 $y(x) = y_h + y_p = (C_1 + C_2 x)e^x - \dfrac{4}{25}\sin3x + \dfrac{3}{25}\cos3x$