

Find the general solution $x^3 \frac{dy}{dx} + x^2 y = 2y^{-4/3}$. [106 中興電機乙丙丁光電 8(1)]

[解]原式 $\Rightarrow y' + x^{-1}y = 2x^{-3}y^{-4/3} \Rightarrow y^{4/3}y' + x^{-1}y^{7/3} = 2x^{-3} \dots\dots\dots(i)$

令 $u = y^{7/3} \Rightarrow u' = 7/3 y^{4/3} y'$, 代入(i)式

$3/7 u' + x^{-1}u = 2x^{-3} \Rightarrow u' + 7x^{-1}u/3 = 14x^{-3}/3 \dots\dots\dots(i)$

$F = e^{\int 7x^{-1}/3 dx} = e^{7/3 \ln x} = x^{7/3}$

$u = \frac{1}{F} [\int F \cdot 14x^{-3} / 3 dx + C] = \frac{1}{x^{7/3}} [\int x^{7/3} \cdot 14x^{-3} / 3 dx + C]$

$= x^{-7/3} [14x^{1/3} + C] = 14x^{-2} + Cx^{-7/3}$

$y^{7/3} = 14x^{-2} + Cx^{-7/3}$