Find the solution $y' - 2xy = x^2 + y^2$ [106 中興精密 7(b)]

[解]原式 $\Rightarrow y' = (x+y)^2 \cdots\cdots(i)$，令 $u = x+y \Rightarrow u' = 1+y' \Rightarrow y' = u'-1$

$(i) \Rightarrow u'-1 = u^2 \Rightarrow \dfrac{du}{dx} = 1+u^2 \Rightarrow \dfrac{du}{1+u^2} = dx \Rightarrow \int \dfrac{du}{1+u^2} = \int dx + C$

$\tan^{-1} u = x + C \Rightarrow u = \tan(x+C) \Rightarrow x+y = \tan(x+C)$