Solve the differential equation $x^2y' = y^2 + xy$. [106 中正機械甲乙丙 1(a)]

[解]令 $y = ux \Rightarrow y' = xu' + u$，原式為

$$x^2(xu' + u) = (ux)^2 + x(ux) \Rightarrow xu' + u = u^2 + u$$

$$x\frac{du}{dx} = u^2 \Rightarrow \frac{du}{u^2} = \frac{dx}{x} \Rightarrow \int \frac{du}{u^2} = \int \frac{dx}{x} + C \Rightarrow -\frac{1}{u} = \ln x + C \Rightarrow -\frac{x}{y} = \ln x + C$$