

求解  $xyy' = x^2 - y^2$ 。 [105 高第一環安 1]

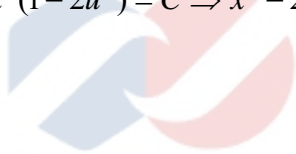
[解] 令  $y = ux \Rightarrow y' = xu' + u$ ，原式為

$$x(ux)(xu' + u) = x^2 - (ux)^2 \Rightarrow uxu' + u^2 = 1 - u^2$$

$$ux \frac{du}{dx} = 1 - 2u^2 \Rightarrow \frac{udu}{1 - 2u^2} = \frac{dx}{x} \Rightarrow \int \frac{udu}{1 - 2u^2} = \int \frac{dx}{x} + k \Rightarrow -\frac{1}{4} \ln(1 - 2u^2) = \ln x + k$$

$$4 \ln x + \ln(1 - 2u^2) = -4k \Rightarrow \ln[x^4(1 - 2u^2)] = -4k \Rightarrow x^4(1 - 2u^2) = e^{-4k}$$

$$x^4(1 - 2u^2) = C \Rightarrow x^4 - 2x^2(ux)^2 = C \Rightarrow x^4 - 2x^2y^2 = C$$



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