

The ODE equation:  $y^2 dx + (1 + xy) dy = 0$ . (a) Verify the ODE is not exact. (b) Find the integrating factor. (c) Find the solution of the ODE. [105 雲科大電子 2]

$$[\text{解}] M = y^2, N = 1 + xy \Rightarrow \frac{\partial M}{\partial y} = 2y, \frac{\partial N}{\partial x} = y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{2y - y}{-y^2} = -\frac{1}{y}$$

$$\mu = e^{\int \frac{1}{y} dy} = e^{-\ln y} = y^{-1}$$

$$u = \int_x \mu M dx + f(y) = \int_x y^{-1} \cdot y^2 dx + f(y) = xy + f(y)$$

$$\frac{\partial u}{\partial y} = \mu N \Rightarrow x + f'(y) = y^{-1} \cdot (1 + xy) \Rightarrow f'(y) = y^{-1} \Rightarrow f(y) = \ln y$$

解為  $xy + \ln y = C$