

Consider the 2<sup>nd</sup> order ordinary differential equation (O.D.E.)  $y'' + 6y' + 9y = \cos 3x$ , and  $y$  is a function of  $x$ . Please find the general solution of the O.D.E. [105 雲科大機械 1]

[解]特徵方程式為

$$\lambda^2 + 6\lambda + 9 = 0 \Rightarrow \lambda = -3, -3$$

$$y_h(x) = (C_1 + C_2x)e^{-3x}$$

$$\text{Let } y_p(x) = A\cos 3x + B\sin 3x \Rightarrow y_p' = -3A\sin 3x + 3B\cos 3x \Rightarrow y_p'' = -9A\cos 3x - 9B\sin 3x$$

代入原式得

$$(-9A\cos 3x - 9B\sin 3x) + 6(-3A\sin 3x + 3B\cos 3x) + 9(A\cos 3x + B\sin 3x) = \cos 3x$$

$$18B\cos 3x - 18A\sin 3x = \cos 3x \Rightarrow A = 0, B = \frac{1}{18}$$

$$\text{解為 } y(x) = y_h(x) + y_p(x) = (C_1 + C_2x)e^{-3x} + \frac{1}{18}\sin 3x$$