

試求解微分方程式 $2t \frac{dy}{dt} - y = 2y^2$; $y(1) = -1$, 其中 $y = y(t)$ 。 [105 海洋造船 3]

[解]原式 $\Rightarrow y^{-2} \frac{dy}{dt} - \frac{1}{2t} y^{-1} = \frac{1}{t}$ (i)

令 $u = y^{-1} \Rightarrow \frac{du}{dt} = -y^{-2} \frac{dy}{dt}$

(i) $\Rightarrow -\frac{du}{dt} - \frac{1}{2t} u = \frac{1}{t} \Rightarrow \frac{du}{dt} + \frac{1}{2t} u = -\frac{1}{t}$

$F = e^{\int \frac{1}{2t} dt} = e^{\frac{1}{2} \ln t} = t^{\frac{1}{2}}$

$u = \frac{1}{F} [\int F \cdot (-\frac{1}{t}) dt + C] = t^{-\frac{1}{2}} [\int t^{\frac{1}{2}} \cdot (-\frac{1}{t}) dt + C] = t^{-\frac{1}{2}} [(-2t^{\frac{1}{2}}) + C] = Ct^{-\frac{1}{2}} - 2$

$y = \frac{1}{Ct^{-\frac{1}{2}} - 2}$

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