

Please find the general solution of the differential equation $(3x^2y + 6xy + \frac{1}{2}y^2)dx + (3x^2 + y)dy = 0$.

[104 高第一先進製造 2]

$$[\text{解}] M = 3x^2y + 6xy + \frac{1}{2}y^2, N = 3x^2 + y \Rightarrow \frac{\partial M}{\partial y} = 3x^2 + 6x + y, \frac{\partial N}{\partial x} = 6x$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{(3x^2 + 6x + y) - 6x}{3x^2 + y} = 1$$

$$\mu = e^{\int 1 dx} = e^x$$

$$u = \int_x \mu M dx + f(y) = \int_x e^x (3x^2y + 6xy + \frac{1}{2}y^2) dx + f(y) \\ = 3(x^2ye^x - \int_x e^x \cdot 6xy dx) + \int_x e^x \cdot 6xy dx + e^x \cdot \frac{1}{2}y^2 + f(y)$$

$$= e^x (3x^2y + \frac{1}{2}y^2) + f(y)$$

$$\frac{\partial u}{\partial y} = \mu N \Rightarrow e^x (3x^2 + y) + f'(y) = e^x (3x^2 + y) \Rightarrow f'(y) = 0 \Rightarrow f(y) = 0$$

$$\text{解為 } e^x (3x^2y + \frac{1}{2}y^2) = C$$