

Solve $(xe^y - 1)dy + (e^{x+y} + ye^y)dx = 0$. [104 高應大模具 1]

$$[\text{解}] M = e^{x+y} + ye^y, N = xe^y - 1 \Rightarrow \frac{\partial M}{\partial y} = e^{x+y} + e^y + ye^y, \frac{\partial N}{\partial x} = e^y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{e^{x+y} + ye^y}{-(e^{x+y} + ye^y)} = -1$$

$$\mu = e^{\int -1 dy} = e^{-y}$$

$$u = \int_x \mu M dx + f(y) = \int_x e^{-y} (e^{x+y} + ye^y) dx + f(y) \\ = e^x + xy + f(y)$$

$$\frac{\partial u}{\partial y} = \mu N \Rightarrow x + f'(y) = e^{-y} \cdot (xe^y - 1) \Rightarrow f'(y) = -e^{-y} \Rightarrow f(y) = e^{-y}$$

$$\text{解為 } e^x + xy + e^{-y} = C$$