Given the equation $y'' - y' - 6y = 8\cos 2x + 6e^{4x}$, find the general solution. [104 雲科大電子 2]

[解]特徵方程式$\lambda^2 - \lambda - 6 = 0 \Rightarrow (\lambda + 2)(\lambda - 3) = 0 \Rightarrow \lambda = -2, 3$

$$y_h = C_1 e^{-2x} + C_2 e^{3x}$$

Let $y_p = (A\cos 2x + B\sin 2x) + Ce^{4x} \Rightarrow y_p' = -2A\sin 2x + 2B\cos 2x + 4Ce^{4x}$

$$y_p'' = -4A\cos 2x - 4B\sin 2x + 16Ce^{4x}$$

代入原式

$$(-4A\cos 2x - 4B\sin 2x + 16Ce^{4x}) - (-2A\sin 2x + 2B\cos 2x + 4Ce^{4x}) - 6(A\cos 2x + B\sin 2x + Ce^{4x}) = 8\cos 2x + 6e^{4x}$$

$$(-10A - 2B)\cos 2x + (2A - 10B)\sin 2x + 6Ce^{4x} = 8\cos 2x + 6e^{4x}$$

$$\begin{cases} -10A - 2B = 8 \\ 2A - 10B = 0 \\ 6C = 6 \end{cases} \Rightarrow A = -\frac{10}{13}, B = -\frac{2}{13}, C = 1$$

$$y = y_h + y_p = C_1 e^{-2x} + C_2 e^{3x} - \frac{10}{13}\cos 2x - \frac{2}{13}\sin 2x + e^{4x}$$