求解 $y'+\frac{1}{x}y=3x^2y^3$。[104 聯合環安 3]

[解]原式 $\Rightarrow y^{-3}y'+\frac{1}{x}y^{-2}=3x^2\cdots\cdots\cdots(\text{i})$

令 $u=y^{-2}\Rightarrow u'=-2y^{-3}y'$

$(\text{i})\Rightarrow -\frac{1}{2}u'+\frac{1}{x}u=3x^2\Rightarrow u'-\frac{2}{x}u=-6x^2$

$F=e^{\int -\frac{2}{x}dx}=e^{-2\ln x}=x^{-2}$

$u=\frac{1}{x^{-2}}[\int x^{-2}\cdot(-6x^2)dx+C]=x^2[(-6x)+C]=-6x^3+Cx^2$

$y^{-2}=-6x^3+Cx^2$