

Solve the differential equation  $(10 - 6y + e^{-3x})dx - 2dy = 0$ . [104 清大生醫丙 2(a)]

$$[\text{解}] M = 10 - 6y + e^{-3x}, N = -2 \Rightarrow \frac{\partial M}{\partial y} = -6, \frac{\partial N}{\partial x} = 0$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-6 - 0}{-2} = 3$$

$$\mu = e^{\int 3dx} = e^{3x}$$

$$u = \int_x \mu M dx + f(y) = \int_x e^{3x} (10 - 6y + e^{-3x}) dx + f(y) \\ = \frac{10}{3} e^{3x} - 2ye^{3x} + x + f(y)$$

$$\frac{\partial u}{\partial y} = \mu N \Rightarrow -2e^{3x} + f'(y) = e^{3x}(-2) \Rightarrow f'(y) = 0 \Rightarrow f(y) = 0$$

$$\text{解為 } \frac{10}{3} e^{3x} - 2ye^{3x} + x = C$$