Solve for $y' + \dfrac{x^2 + y^2 + x}{2xy} = 0$. [104 清大核工工科 3]

[解]原式 $\Rightarrow (x^2 + y^2 + x)dx + 2xydy = 0$

$M = x^2 + y^2 + x \Rightarrow \dfrac{\partial M}{\partial y} = 2y$

$N = 2xy \Rightarrow \dfrac{\partial N}{\partial x} = 2y$

$\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} \Rightarrow$ 原式為正合微分方程式

$u = \displaystyle\int_x Mdx + f(y) = \int_x (x^2 + y^2 + x)dx + f(y) = \dfrac{x^3}{3} + xy^2 + \dfrac{x^2}{2} + f(y)$

$\dfrac{\partial u}{\partial y} = N \Rightarrow 2xy + f'(y) = 2xy \Rightarrow f'(y) = 0 \Rightarrow f(y) = 0$

解為 $u = C \Rightarrow \dfrac{x^3}{3} + xy^2 + \dfrac{x^2}{2} = C$