

Solve the differential equation $x^2y' - xy = y^2$. [104 淡江電機 4]

[解]原式 $\Rightarrow y^{-2}y' - \frac{1}{x}y^{-1} = \frac{1}{x^2} \dots\dots\dots$ (i)

令 $u = y^{-1} \Rightarrow u' = -y^{-2}y'$

(i) $\Rightarrow -u' - \frac{1}{x}u = \frac{1}{x^2} \Rightarrow u' + \frac{1}{x}u = -\frac{1}{x^2}$

$F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$u = \frac{1}{x} \left[\int x \cdot \left(-\frac{1}{x^2}\right) dx + C \right] = \frac{1}{x} [-\ln x + C] = \frac{C - \ln x}{x}$

$y = u^{-1} = \frac{x}{C - \ln x}$

