Solve $x^2y'' - 2x(x+1)y' + 2(x+1)y = 0$. One solution is $y = x$. [104 暨南電機 5]

[解]令 $y = vx \Rightarrow y' = v'x + v, \; y'' = v''x + 2v'$

原式 $\Rightarrow x^2(v''x + 2v') - 2x(x+1)(v'x + v) + 2(x+1)vx = 0$

$x^3v'' + [2x^2 - 2x^2(x+1)]v' + [-2x(x+1) + 2x(x+1)]v = 0$

$x^3v'' - 2x^3v' = 0 \Rightarrow v'' - 2v' = 0$ 這是 v' 的一階線性

$v' = ke^{\int 2 \cdot dx} = ke^{2x}$

$v = \int ke^{2x}dx + C_2 = C_1e^{2x} + C_2$

$y = vx = C_1xe^{2x} + C_2x$