

Solve $x^2y'' - 2x(x+1)y' + 2(x+1)y = 0$. One solution is $y = x$. [104 暨南電機 5]

[解] 令 $y = vx \Rightarrow y' = v'x + v, y'' = v''x + 2v'$

$$\text{原式} \Rightarrow x^2(v''x + 2v') - 2x(x+1)(v'x + v) + 2(x+1)vx = 0$$

$$x^3v'' + [2x^2 - 2x^2(x+1)]v' + [-2x(x+1) + 2x(x+1)]v = 0$$

$$x^3v'' - 2x^3v' = 0 \Rightarrow v'' - 2v' = 0 \text{ 這是 } v' \text{ 的一階線性}$$

$$v' = ke^{\int 2 dx} = ke^{2x}$$

$$v = \int ke^{2x} dx + C_2 = C_1 e^{2x} + C_2$$

$$y = vx = C_1 x e^{2x} + C_2 x$$

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